International Journal of Theoretical Physics, Vol. 25, No. 5, 1986

Quantum de Sitter Fiber Bundle Interpretation of Hadron Extension

R. R. Aldinger[1]

Received September 1, 1985

A quantized geometrical description of hadron extension is developed based on Drechsler's soldered de Sitter fiber bundle, which possesses as fiber a local four-dimensional Riemannian space of constant curvature characterized by a radius of curvature R of the order of 1 Fermi. The structural (gauge) group of the bundle is a de Sitter SO(4, 1), which contains all observable transformations (rotations as well as translations). The quantized de Sitter-structured connection of the bundle leads to a set of self-interaction gauge operators that "act back" on the de Sitter bundle by inducing a local curvature, which, in turn, affects a small neighborhood of the adjoining space-time position, leading to the experimentally observed "size" of the hadron. A particular choice for the quantized Lorentz cross section (gauge) is made that leads to the mathematically consistent and experimentally verifiable hadron model, the quantum relativistic rotator. Also investigated is the limit corresponding to taking the radius of curvature of the de Sitter fiber to infinity.

1. INTRODUCTION

Much attention has been directed toward obtaining a clear understanding of the internal structure of hadrons through the use of quantum field-theoretic methods, all of which employ the dynamics of certain more fundamental subunits, which are taken to constitute the experimentally observed hadronic bound states (Gell-Mann, 1962; Chodos *et al.*, 1974; Salam and Strathdee, 1977, 1978; Marciano and Pagels, 1978). The basic shortcomings of these particular approaches is that they do not determine in terms of the assumed underlying dynamics precisely what field structures are responsible for the observed nonlocality of hadrons and they fail to explain adequately the apparent confinement mechanism. In short, the methods of quantum chromodynamics have not sufficiently determined the complete low-energy structure of the theory.

[1]Department of Physics, Eastern Illinois University, Charleston, Illinois 61920.

0020-7748/86/0500-0527$05.00/0

In this paper the problems concerning the internal structure of hadrons are addressed from a completely different point of view. In particular, the experimentally observed nonlocal character of hadrons is accounted for by borrowing from general relativity the fundamental concept that it is the distribution of matter itself that determines the geometry of the underlying space–time structure. I therefore assume that in the same way that the long-range gravitational interactions lead to a cosmological curvature, the short-range strong interactions may also lead to a "local curling," but on a much smaller microscopic scale.

The basic idea is to endow ordinary space–time with a richer structure than that implied by relativity by attaching to each position x an internal space characterized by an elementary length parameter of the order of 1 Fermi, thereby allowing for additional internal degrees of freedom. As a consequence of the presence of a microscopic distribution of matter at some position x, these additional degrees of freedom induce locally a nonzero curvature of the (hadron's) internal space, which leads to a microscopic "curling" of the immediate neighborhood of the adjoining physical space–time position. The actual manifestation of the local nonzero curvature of the internal space is attributed to the presence of a set of strong (self-interaction) compensating fields, which provides a non-Abelian gauge-type description of hadron extension, thereby avoiding the difficulties associated with the determination of the dynamics of more elementary constituents, which may be "trapped" within the matter distribution.

The mathematical structure on which a gauge description of hadron extension is formulated is a fiber bundle $E(B, F, G, P)$ constructed over base manifold B, possessing fiber F, and associated with the principal fiber bundle $P = P(B, G)$ (Kobayashi and Nomizu, 1963). The structural (gauge) group G of the fiber plays the role of a dynamical group[2] (Barut and Bohm, 1965), which determines the internal motion of the hadron (degrees of freedom belonging to the fibers).

The fiber bundle formalism of hadron extension presented here is well-suited for the inclusion of gravity simply by choosing as base manifold an appropriately curved Riemannian space–time V_4. However, in order to concentrate specifically on how the short-range strong-interaction gauge fields affect the underlying base manifold, I assume that any long-range gravitational perturbations are locally negligible and therefore choose as base manifold flat Minkowski space–time, $B = M_4$.

In order to allow for hadronic internal degrees of freedom, an elementary length parameter, related to the range of the self-interaction gauge fields, is built directly into the mathematical structure by choosing as fiber

[2]By dynamical group I mean a symmetry group for a quantum mechanical system possessing a mass-spin spectrum.

a four-dimensional pseudo-Riemannian space V_4' of constant curvature with intrinsic radius $R \approx 1$ Fermi. Since the measurement of distance can be made only in Minkowski space, the intrinsic radius of the internal space V_4' does not have any particular physical relevance. Therefore, in order to transfer a physical meaning to the intrinsic curvature of V_4' I choose a fiber bundle with Cartan connection (Drechsler and Mayer, 1977), which is characterized by the following: (1) the structural group G of the fiber bundlle $E(B, F, G, P)$ associated with $P = P(B, G)$ acts transitively on F, i.e., F is the homogeneous space G/G', where G' is the stability subgroup (residual gauge group) of G leaving the point $\mathring{\xi} \in F$ fixed;[3] (2) $\dim F = \dim B = n$; and (3) the tangent spaces $T_x(B)$ and $T_{\mathring{\xi}}(V_4'(x))$ are isomorphic to one another, implying that the local fiber F_x over $x \in B$ is "soldered" to B in such a way that F_x is tangent to B at $x \equiv \mathring{\xi}$ for any $x \in B$. For the fiber bundle chosen here (in the absence of gravitational perturbations) the "soldering" is obtained by identifying a Minkowski subspace of the fiber bundle with the local tangent space of M_4, which thereby transfers the physical meaning of distance directly to the fiber so that R may be measured in centimeters.

In order to translate the purely mathematical formalism of a "soldered" fiber bundle constructed over space-time into a physically acceptable theory, it is necessary to decide on what to use for the structural (gauge) group G. Motivated by the facts that (1) the inclusion of translations into the structural group should lead to a de Sitter-structured fiber bundle (Smrz, 1983), (2) the pseudo-Riemannian noncompact coset space $V_4' = SO(4, 1)/SO(3, 1)$ contains the stability subgroup $SO(3, 1)$, which, via the soldering mechanism, may be identified with the physical Lorentz subgroup of the Poincaré group (thereby supplying an important connection between the external space-time transformations and the internal motions), and (3) a de Sitter $SO(4, 1)$ plays the central role in the development of the model of the quantum relativistic rotator (QRR) (Aldinger *et al.*, 1983; Bohm *et al.*, 1983) in that the second-order Casimir operator of an $SO(4, 1)$ leads to an experimentally verifiable rotator-like mass-spin spectrum (Aldinger *et al.*, 1984), one is led to choose for the structural group G a de Sitter $SO(4, 1)$. Therefore the physically relevant bundle used for a geometrical (gauge) description of hadron extension is a de Sitter fiber bundle with Cartan connection,

$$T^R(M_4) \approx E(M_4, SO(4, 1)/SO(3, 1), SO(4, 1), P)$$

where $P = P(M_4, SO(4, 1))$, hereafter referred to as Drechsler's de Sitter fiber bundle (Drechsler, 1975, 1977a, b).

[3] $\mathring{\xi}$ is the origin of the homogeneous coset space G/G' and is the point of contact between base space and fiber.

The connection between a hadronic matter distribution and the underlying fiber bundle geometry is provided for by coupling, in the usual way, the curvature of the bundle space to the hadronic current, which itself is derived from the matter fields, $\psi(x, \xi)$, where x determines the external space-time position only and ξ labels positions along the fibers. However, in this paper I am primarily concerned with the group-theoretic aspects associated with a gauge-type description for the extension of isolated (noninteracting) hadrons and will not invesigate hadronic matter fields and how they relate to the quantities defining the geometry in the underlying bundle space.[4]

2. $SO(4,1)$ CONNECTION AND CURVATURE

In this section I give a brief review of Drechsler's de Sitter fiber bundle and analyze the decomposition of the $SO(4,1)$ connection and corresponding curvature expressions.[5] The physically important concept of "soldering" and the interpretation of the gauge translations will be discussed in the following section.

The mathematical structure leading to a geometrical description of hadron extension is provided for by Drechsler's de Sitter fiber bundle

$$T^R(M_4) \approx E(M_4, SO(4,1)/SO(3,1), SO(4,1), P)$$

where the principal bundle $P = P(M_4, SO(4,1))$. The standard fiber of $T^R(M_4)$ is a four-dimensional pseudo-Riemannian space V'_4 which is isomorphic to the homogeneous noncompact coset space $SO(4,1)/SO(3,1)$ (i.e., a de Sitter space). The structural group of $T^R(M_4)$ is a de Sitter $SO(4,1)$ [which acts as the dynamical group of motion along the fibers of $T^R(M_4)$], which possesses an $SO(3,1)$ as the (noncompact) stability subgroup.

The de Sitter space V'_4 on which $SO(4,1)$ acts as the symmetry group of motion may be embedded into a five-dimensional pseudo-Euclidean space and is specified by a four-dimensional hypersurface which is noncompact in time and compact in the space directions. In terms of coordinates of the pseudo-Euclidean space, the hypersurface may be expressed as

$$\xi^A \xi_A = \xi^A \xi^B \eta_{AB} = -R^2, \qquad A, B = i, j, 4; \quad i, j = 0, 1, 2, 3 \tag{1}$$

where the de Sitter metric $\eta_{AB} = \text{diag}(1, -1, -1, -1, -1)$ and R is a fixed length parameter characterizing the radius of the space.

[4]For a discussion of how spinor-valued matter fields act as a source for the geometric curvature fields of the bundle space, see Drechsler (1975).
[5]For further details see Drechsler and Mayer (1977).

The de Sitter group is the group of transformations that leaves the quadratic form of equation (1) invariant, i.e., it is the group of motion in V_4'. A basis for the Lie algebra of $G = SO(4,1)$ is given by

$$J_{AB} = -J_{BA} \tag{2}$$

which obey the commutation relations

$$[J_{AB}, J_{CD}] = -i(\eta_{AC}J_{BD} + \eta_{BD}J_{AC} - \eta_{AD}J_{BC} - \eta_{BC}J_{AD}) \tag{3}$$

In order to exhibit the subgroup structure of $SO(4,1)$ I now define

$$\pi_i \equiv \frac{1}{R} J_{4i}, \qquad i = 0, 1, 2, 3 \tag{4}$$

where J_{4i} generate de Sitter rotations in the $4-i$ plane (pseudorotations for $i=0$) and π_i are the corresponding generators of translation along the de Sitter fibers (the de Sitter boosts). Therefore the commutation relations of equation (3) take on the form $[\eta_{ij} = \mathrm{diag}(1, -1, -1, -1)]$

$$[J_{ij}, J_{kl}] = -i(\eta_{ik}J_{jl} + \eta_{jl}J_{ik} - \eta_{il}J_{jk} - \eta_{jk}J_{il}) \tag{5a}$$

$$[\pi_i, J_{jk}] = -i(\eta_{ik}\pi_j - \eta_{ij}\pi_k) \tag{5b}$$

$$[\pi_i, \pi_j] = i\frac{1}{R^2} J_{ij} \tag{5c}$$

These relations display the $SO(3,1)_{J_{ij}}$ stability subalgebra (spanned by the J_{ij} that generate rotations around the ξ^4 axis leaving the point $\mathring{\xi}$ fixed[6]) together with the vector subspace spanned by the π_i (the de Sitter boosts). I am using the convention that indices $A, B, C, D, \ldots$ (running over 0, 1, 2, 3, 4) and indices $i, j, k, l, \ldots$ (running over 0, 1, 2, 3) belong to the internal space (the de Sitter fibers), whereas indices $\mu, \nu, \rho, \sigma, \ldots$ (running over 0, 1, 2, 3) belong to the external base space.

The de Sitter-structured connection in $P(M_4, SO(4,1))$ is a matrix-value four-vector field on Minkowski space with matrices defining a representation of the Lie algebra of $SO(4,1)$ and has the form

$$\Gamma_\mu^R(x) = \tfrac{1}{2}\Gamma_\mu^{AB}(x)J_{AB} \tag{6}$$

where $\Gamma_\mu^R(x) = h_\mu^i(x)\Gamma_i^R(x)$, with $h_\mu^i(x)$ the space-time tetrad fields. The ten generators J_{AB} carry the dependence on the group coordinates and satisfy equation (3), while the strong interaction gauge potentials $\Gamma_\mu^{AB}(x)$ depend only on the Minkowski coordinates and represent the 40 coefficients of an $SO(4,1)$ connection that determines the nature of the local frames on $T^R(M_4)$ in going from a point $x \in M_4$ to an infinitesimally close point, i.e., they describe the observable change in the frame.

[6] $\mathring{\xi}^A$ is given by $\mathring{\xi}^A = (0, 0, 0, 0, -R)$ and denotes the coordinates of the contact point between base space and fiber.

The horizontal lift of a tangent vector ∂_μ at $x \in M_4$ to an arbitrary point $p \in P(M_4, SO(4,1))$ is

$$D_\mu = \partial_\mu + i\Gamma^R_\mu(x) \tag{7}$$

and the curvature of the de Sitter connection is obtained by taking the commutator of two horizontal vector fields:

$$[D_\mu, D_\nu] = i\mathcal{R}^R_{\mu\nu}(x) \tag{8}$$

where

$$\mathcal{R}^R_{\mu\nu}(x) \equiv \partial_\mu \Gamma^R_\nu(x) - \partial_\nu \Gamma^R_\mu(x) + i[\Gamma^R_\mu(x), \Gamma^R_\nu(x)] \tag{9}$$

The Lie algebra $\mathcal{G}$ of G may be decomposed according to

$$\mathcal{G} = \mathcal{L} \oplus \mathcal{T} \tag{10}$$

where $\mathcal{L}$ is the Lie algebra of the stability subgroup $SO(3,1)_{J_{ij}}$ and $\mathcal{T}$ is a four-dimensional vector subspace of $\mathcal{G}$ spanning the tangent space to V_4' at $\mathring{\xi}$. According to this decomposition, the $SO(4,1)$ connection may be expressed in the following way:

$$\Gamma^R_\mu(x) = \Gamma^l_\mu(x) + \Gamma^t_\mu(x) \tag{11}$$

$$= \tfrac{1}{2}\Gamma^{ij}_\mu(x) J_{ij} + \Gamma^{4i}_\mu(x) J_{4i} \tag{12}$$

where the de Sitter (R-valued) connection has been decomposed into $SO(3,1)_{J_{ij}}$ (l-valued) and de Sitter boost (t-valued) components.

Using the decomposition of equation (11), one finds for the horizontal lift

$$D_\mu = \partial_\mu + i\Gamma^l_\mu + i\Gamma^t_\mu \tag{13}$$

(where for the sake of brevity I have suppressed the x dependence). Therefore

$$[D_\mu, D_\nu] = i\mathcal{R}^R_{\mu\nu} = iQ^l_{\mu\nu} + iS^t_{\mu\nu} \tag{14}$$

where

$$Q^l_{\mu\nu} \equiv R^l_{\mu\nu} + i[\Gamma^t_\mu, \Gamma^t_\nu] \tag{15a}$$

$$R^l_{\mu\nu} \equiv \partial_\mu \Gamma^l_\nu - \partial_\nu \Gamma^l_\mu + i[\Gamma^l_\mu, \Gamma^l_\nu] \tag{15b}$$

$$S^t_{\mu\nu} \equiv \partial_\mu \Gamma^t_\nu - \partial_\nu \Gamma^t_\mu + i[\Gamma^l_\mu, \Gamma^t_\nu] + i[\Gamma^t_\mu, \Gamma^l_\nu] \tag{15c}$$

The l-valued component of the de Sitter curvature is given by equation (15a), where $R^l_{\mu\nu}$ is the $SO(3,1)$-structured curvature and the second term is the contribution to $Q^l_{\mu\nu}$ originating from the translational components Γ^t_μ, whereas $S^t_{\mu\nu}$ is the translational (t-valued) component of the de Sitter curvature and corresponds to torsion.

Notice that a dimensional coupling constant was not explicitly written in front of the $\Gamma_\mu^R(x)$ terms on the right-hand side of equations (7) and (11). Such a strong-interaction coupling is assumed to be absorbed in the $\Gamma_\mu^R(x)$, which therefore carry the dimension of inverse length. One could always replace the $\Gamma_\mu^R(x)$ with $g\hat{\Gamma}_\mu^R(x)$, where g is a constant with the dimension of inverse length and $\hat{\Gamma}_\mu^R(x)$ is the dimensionless analogue of the de Sitter connection. Then a factor of g will appear on the right-hand side of equation (14) as well as in front of the commutation relations (15), leading to the interpretation of these relations as the gauge field "self-interactions" (Wheeler, 1962).

Choosing a gauge [cross section in $P(M_4, SO(4,1))$] leads to gauge-fixing relations (constraints on the gauge potentials), which are used to eliminate any unphysical degrees of freedom in the formalism. From these gauge constraints it follows that certain terms are absent from the set of expressions making up the de Sitter curvature field, (15). But the choice of a particular gauge requires physical motivation and cannot be imposed strictly from the mathematics alone. In Section 4 I apply the mathematical formalism of the quantum analogue of Drechsler's soldered de Sitter fiber bundle to the physical model of the QRR, which will require a set of strong constraints imposed on the (quantized) $SO(4,1)$ connection, resulting in a major simplification of the (quantized) curvature expression given by (15).

I conclude this section by noting that the $SO(4,1)$ curvature fields $\mathcal{R}_{\mu\nu}^R(x)$ are not independent, but satisfy the kinematic constraint (Bianchi identities)

$$D_\mu \mathcal{R}_{\rho\sigma}^R(x) + D_\rho \mathcal{R}_{\sigma\mu}^R(x) + D_\sigma \mathcal{R}_{\mu\rho}^R(x) = 0 \tag{16}$$

which is a direct consequence of the Jacobi identity for the horizontal lift:

$$[D_\mu, [D_\rho, D_\sigma]] + [D_\rho, [D_\sigma, D_\mu]] + [D_\sigma, [D_\mu, D_\rho]] = 0 \tag{17}$$

3. SOLDERING AND GAUGE TRANSLATIONS

In the previous section I reviewed Drechsler's de Sitter fiber bundle $T^R(M_4)$ constructed over Minkowski space-time and possessing as fiber the noncompact coset space $V_4' = SO(4,1)/SO(3,1)$ on which an $SO(4,1)$ acts as the (internal) symmetry group of motion. The $SO(4,1)$ connection was found to decompose according to

$$\Gamma_\mu^R(x) = \Gamma_\mu^l(x) + \Gamma_\mu^t(x) \tag{18}$$

where

$$\Gamma_\mu^l(x) = \tfrac{1}{2}\Gamma_\mu^{ij}(x) J_{ij} \tag{19a}$$

$$\Gamma^{t}_{\mu}(x)=\Gamma^{4i}_{\mu}(x)J_{4i} \tag{19b}$$

The four-vector gauge potentials $\Gamma^{4i}_{\mu}(x)$ are connected with the group of vertical translations along the de Sitter fibers, but may be associated with "real" translations as carried out in Minkowski space-time through the additional concept of the "soldering" mechanism.

It has been shown that the Lie algebra $\mathcal{G}$ of $G=SO(4,1)$ decomposes as $\mathcal{G}=\mathcal{L}\oplus\mathcal{T}$, where $\mathcal{L}$ is the Lie algebra of the $SO(3,1)_{J_{ij}}$ stability subgroup of $SO(4,1)_{J_{ij}\pi_i}$ and $\mathcal{T}$ is the four-dimensional vector subspace of $\mathcal{G}$ corresponding to (vertical) translations. Let $\Gamma^R(x)$ be an $SO(4,1)$-valued connection form of a connection in $P=P(M_4, SO(4,1))$ which, according to equation (10), decomposes into l- and t-valued components:

$$\Gamma^{R}(x)=\Gamma^{l}(x)+\Gamma^{t}(x) \tag{20}$$

For some chosen gauge (cross section) in P there exists a subbundle $P'=P'(M_4\ SO(3,1))$ of P in which the fibers are determined by the residual gauge group $SO(3,1)$.[7] On the subbundle P', $\Gamma^l(x)$ plays the role of the l-valued connection form, while $\Gamma^t(x)$ is an R^4-valued 1-form which may be identified as the soldered canonical form $\theta(X_\mu)$,[8] where X_μ is a tangent vector at some $u(x)\in$ F (Trautman, 1970; Smrz, 1977; Drechsler, 1977b).[9] That is, the principal fiber bundle P is mapped onto a subbundle P' in such a way that the six $SO(3,1)_{J_{ij}}$ gauge potentials $\Gamma^{ij}_{\mu}(x)$ define a connection (l-valued) on P', while the four gauge potentials $\Gamma^{4i}_{\mu}(x)$ provide the "soldering" that makes P' the bundle of linear frames (i.e., the bundle of all bases of tangent vector spaces) of M_4 and thereby identifies points in the fiber bundle with the local frames of M_4. Thereby a dynamical significance has been attached to the internal coordinates ξ, since making a change in the external (base manifold) position x forces, via the soldering mechanism, a corresponding transformation of ξ along the locally "attached" fiber F_x over $x\in M_4$.[10] Compare this formulation with the conventional non-Abelian gauge field theories (Yang and Mills, 1954), which are described by general fiber bundles that are "loosely" connected (unsoldered) to the base manifold, where only the direction of the axis of the local internal space has any particular significance.

[7] The principal fiber bundle P' is a subspace of P obtained by restricting the homeomorphisms $F\to F_x$ (where F_x is the local fiber over $x\in B$) of P in such a way that $\mathring{\xi}\in F$ is always mapped into the point of contact of fiber and base space at $x\in B$ (Drechsler, 1977b).

[8] This identification holds since the R^4-valued 1-form $\Gamma^t(x)$ has the same transformation properties as the canonical form, which follows as a consequence of the fact that in $SO(4,1)$ the adjoint map adg', $g'\in SO(3,1)$, acts on the vector subspace $\mathcal{T}$ as the four-dimensional real representation of $SO(3,1)$. See Smrz (1977) and Drechsler (1977b).

[9] If X_x is the projection of X_μ at $x\in B$, then the canonical form $\theta(X_\mu)\in R^4$ gives the components of X_x with respect to the linear frame $u(x)$ at x (Smrz, 1977).

[10] Any point $\xi\in V_4'$ can be obtained from $\mathring{\xi}$ by way of the de Sitter boosts.

The identification of the R^4-valued 1-form $\Gamma^t(x)$ as the soldered canonical form $\theta(X_\mu)$ implies that the corresponding gauge potentials $\Gamma^{4i}_\mu(x)$ must be interpreted as proportional to the components of $\theta(X_\mu)$ (i.e., proportional to the space-time tetrads $h^i_\mu(x)$ (Smrz, 1977). I therefore write

$$\Gamma^{4i}_\mu(x) = gh^i_\mu(x) \tag{21}$$

where g is a fundamental constant of proportionality with the dimension of inverse length. The covariant components of the Minkowski metric tensor are given by

$$g_{\mu\nu}(x) = h^i_\mu(x) h^j_\nu(x) \eta_{ij} \tag{22}$$

Using the dimensionless analogue of the $SO(4,1)_{J_{ij}\hat{\pi}}$ generator of translations (de Sitter boosts), where

$$\hat{\pi}_i \equiv J_{4i}, \qquad i = 0, 1, 2, 3 \tag{23}$$

one has that the t-valued connection may be written as

$$\Gamma^t_\mu(x) = gh^i_\mu(x)\hat{\pi}_i \tag{24}$$

Furthermore, since the base space of the bundle $T^R(M_4)$ has been chosen to be locally free from gravitational perturbations (i.e., Minkowskian flat), one has that $h^i_\mu(x) = \delta^i_\mu$ and

$$\Gamma^t_\mu(x) = g\hat{\pi}_\mu \tag{25}$$

This result shows that the translational gauge is automatically fixed as a consequence of the identification of $\Gamma^t_\mu(x)$ with the canonical form, which directly leads to the determination of the gauge potentials $\Gamma^{4i}_\mu(x)$ given by (21). Therefore, we have no freedom in choosing a particular cross section in $\Gamma^t_\mu(x)$ and translations should not be considered as true gauge degrees of freedom.

Another important consequence of the soldering mechanism is that the $SO(3,1)_{J_{ij}}$ stability subgroup of the structural group $SO(4,1)_{J_{ij}\hat{\pi}_i}$ may be identified with the physical Lorentz subgroup $SO(3,1)_{J_{\mu\nu}}$ of the Poincaré group $\mathcal{P}_{J_{\mu\nu}P_\mu}$ (Bohm, 1979). Therefore the l-valued connection given by equation (19a) may be expressed as

$$\Gamma^l_\mu(x) = \tfrac{1}{2}\Gamma^{\rho\sigma}_\mu(x) J_{\rho\sigma} \tag{26}$$

Using this result along with (25) gives for the $SO(4,1)$ connection

$$\Gamma^R_\mu(x) = \tfrac{1}{2}\Gamma^{\rho\sigma}_\mu(x) J_{\rho\sigma} + g\hat{\pi}_\mu \tag{27}$$

where the Lorentz (l-valued) connection coefficients $\Gamma^{\rho\sigma}_\mu(x)$ are classical fields and are functions of the coordinates x_μ. The $J_{\rho\sigma}$ and $\hat{\pi}_\mu$ generate a representation of the structural group $G = SO(4,1)_{J_{\mu\nu}\hat{\pi}_\mu}$, where $J_{\mu\nu}$ generate total angular momentum and fulfill the commutation relations of the Lorentz group:

$$[J_{\mu\nu}, J_{\rho\sigma}] = -i(g_{\mu\rho}J_{\nu\sigma} + g_{\nu\sigma}J_{\mu\rho} - g_{\mu\sigma}J_{\nu\rho} - g_{\nu\rho}J_{\mu\sigma}) \tag{28a}$$

and $\hat{\pi}_\mu$ satisfy

$$[J_{\mu\nu}, \hat{\pi}_\rho] = i(g_{\nu\rho}\hat{\pi}_\mu - g_{\mu\rho}\hat{\pi}_\nu) \tag{28b}$$

$$[\hat{\pi}_\mu, \hat{\pi}_\nu] = iJ_{\mu\nu} \tag{28c}$$

Although no freedom for the possible choices of the translational gauge exists [$\Gamma^t_\mu(x)$ is formally fixed according to equation (25)], the Lorentz gauge is still completely unspecified and a particular gauge choice may be made by determining the l-valued connection coefficients $\Gamma^{\rho\sigma}_\mu(x)$. In the following section, a specific choice for the quantum analogues of $\Gamma^{\rho\sigma}_\mu(x)$ will be made which leads to the physically acceptable model of the QRR.

4. QUANTIZED DE SITTER CONNECTION AND AN EXAMPLE: THE QRR-GAUGE CHOICE

Thus far I have discussed the de Sitter structural group $SO(4,1)_{J_{\mu\nu}\hat{\pi}_\mu}$ and the Poincaré group $\mathcal{P}_{J_{\mu\nu}P_\mu}$ as they pertain to the geometrical formulation of hadron extension. A quantum physical system (hadron) is not described in terms of purely geometrical quantities, but, according to the fundamental postulates of quantum mechanics, by an algebra of operators that act in the space of physical states. According to this interpretation, an elementary particle has as its mathematical image an irreducible representation space of the Poincaré group (Wigner, 1939). Therefore, to arrive at a plausible description of a quantum physical system, all previously discussed geometrical quantities should be represented by their Hermitian counterparts and accordingly the generators $J_{\rho\sigma}$ and $\hat{\pi}_\mu$ of the non-Abelian structure group $SO(4,1)_{J_{\mu\nu}\hat{\pi}_\mu}$ will be understood as Hermitian operators in a unitary representation. Furthermore, in order not only to apply the de Sitter connection to functions of x, but to generalize its applicability such that it acts on vectors of the space of physical states requires the replacement of the classical Lorentz connection coefficients $\Gamma^{\rho\sigma}_\mu(x)$ with their quantum mechanical analogues, thereby treating them as general quantum operators. Therefore equation (27) is replaced with the following operator expression:

$$(\Gamma^R_\mu(x))^{\text{op}} = \tfrac{1}{2}(\tfrac{1}{2}\{(\Gamma^{\rho\sigma}_\mu(x))^{\text{op}}, J_{\rho\sigma}\}) + g\hat{\pi}_\mu \tag{29}$$

where $\{A, B\} = AB + BA$.

A physically interesting formulation is obtained by making the following Lorentz-valued gauge (cross-section) choice (Bohm, 1979):

$$(\Gamma^{\rho\sigma}_\mu(x))^{\text{op}} = g\eta^\rho_\mu \hat{P}^\sigma \tag{30}$$

where $\hat{P}_\mu = P_\mu M^{-1}(=M^{-1}P_\mu)$ and $P_\mu P^\mu = M^2$, where g is the strong-interaction coupling constant with dimension of inverse length. With this particular gauge choice, the $SO(4,1)_{J_{\mu\nu}\hat{\pi}_\mu}$ connection becomes

$$(\Gamma^R_\mu(x))^{\text{op}} = g(\tfrac{1}{4}\{J_{\mu\sigma}, \hat{P}^\sigma\} + \hat{\pi}_\mu) \tag{31}$$

I now make the transition to the physically relevant model of the QRR, whose central feature is motivated by introducing the postulated center operator of Finkelstein (1949):

$$b_\mu = \frac{1}{2M}\{J_{\mu\sigma}, \hat{P}^\sigma\} \tag{32a}$$

or, in dimensionless form where $\hat{b}_\mu = Mb_\mu$ $(=b_\mu M)$,

$$\hat{b}_\mu = \tfrac{1}{2}\{J_{\mu\sigma}, \hat{P}^\sigma\} \tag{32b}$$

which specifies b_μ in terms of the generators of a unitary representation of the Poincaré group $\mathscr{P}_{J_{\mu\nu}P_\mu}$.

Using the well-known commutation relations of the Poincaré group [see (41) below], it is found that $J_{\mu\nu}$ and $\hat{b}_\mu$ generate a Hermitian representation of the Lie algebra of an $SO(4,1)_{J_{\mu\nu}\hat{b}_\mu}$, where

$$[J_{\mu\nu}, J_{\rho\sigma}] = -i(g_{\mu\rho}J_{\nu\sigma} + g_{\nu\sigma}J_{\mu\rho} - g_{\mu\sigma}J_{\nu\rho} - g_{\nu\rho}J_{\mu\sigma}) \tag{33a}$$

$$[J_{\mu\nu}, \hat{b}_\rho] = i(g_{\nu\rho}\hat{b}_\mu - g_{\mu\rho}\hat{b}_\nu) \tag{33b}$$

$$[\hat{b}_\mu, \hat{b}_\nu] = iJ_{\mu\nu} \tag{33c}$$

With this result along with the fact that $b_\mu = \hat{b}_\mu M^{-1}$ $(=M^{-1}\hat{b}_\mu)$ is the QRR's (time-independent) origin operator (Aldinger, 1985) (which specifies the placement of the origin where the QRR's center of mass $Y_\mu = b_\mu + \hat{P}_\mu\tau$), one is justified in making the following identification of the $SO(4,1)_{J_{\mu\nu}\hat{\pi}_\mu}$ dimensionless operator of translations:

$$\hat{\pi}_\mu \to \hat{b}_\mu \tag{34}$$

Therefore the $SO(4,1)_{J_{\mu\nu}\hat{b}_\mu}$-valued connection operator in the "QRR-gauge" takes on the form

$$(\Gamma^R_\mu(x))^{\text{op}} = g((\Gamma^l_\mu(x))^{\text{op}} + (\Gamma^t_\mu(x)^{\text{op}}) \tag{35a}$$

$$= g(\tfrac{1}{2}\hat{b}_\mu + \hat{b}_\mu) \tag{35b}$$

$$= \tfrac{3}{2}g\hat{b}_\mu \tag{35c}$$

where the explicit form of the Finkelstein center operator given in equation (32b) has made it possible to combine the l- and t-valued connection operators into one "generalized" $SO(4,1)_{J_{\mu\nu}\hat{b}_\mu}$-valued connection operator (35c). The $SO(4,1)_{J_{\mu\nu}\hat{b}_\mu}$ gauge coupling constant g characterizes the relative strength of the (strong) interaction between the hadronic matter fields and the gauge potentials. Here I am concerned with the extension of isolated (noninteracting) hadrons and therefore assume that the connection operators $\hat{b}_\mu(x)$ extend their influence to the "boundary" of the micro-de Sitter sphere (hadronic bag), which is determined by the intrinsic radius of the internal space. A direct relationship is therefore expected between the strong coupling constant g and the radius R of the micro-de Sitter space (which, as discussed in Section 5, has the empirical value of $R \approx 1$ Fermi).

In classical gauge field theory the gauge is fixed by imposing some constraint on the gauge potentials $A^a_\mu(x)$, which is then used to eliminate any unphysical degrees of freedom. Common gauge conditions are the Coulomb gauge, where $\nabla \cdot A^a(x) = 0$, and the Lorentz gauge, where $\partial^\mu A^a_\mu(x) = 0$. For the quantum mechanical $SO(4,1)_{J_{\mu\nu}\hat{b}_\mu}$-valued connection in the "QRR-gauge" we have the following operator identity, which is a consequence of equation (32) (Aldinger, 1985; Staunton, 1976):

$$\{\hat{b}_\mu, P^\mu\} = 0 \tag{36a}$$

where

$$P \cdot \hat{b} = \tfrac{3}{2} iM = -\hat{b} \cdot P \tag{36b}$$

This identity follows from the specific form of Finkelstein's center operator and reduces, classcially, to the Lorentz gauge constraint. Note also that as a direct consequence of the form of equation (32) we have for the frame at rest (proper Lorentz frame) where $\hat{P}_\mu = (1, 0, 0, 0)$

$$\hat{b}_0 = 0 \qquad \text{(in the rest frame)} \tag{37}$$

displaying the fact that the $SO(4,1)_{J_{\mu\nu}\hat{b}_\mu}$ generators of de Sitter boosts are space like four-vectors.

One can now express the full quantum mechanical analogue of the $SO(4,1)_{J_{\mu\nu}\hat{b}_\mu}$ horizontal lift [equation (13)] in the "QRR-gauge":

$$B_\mu = P_\mu + \tfrac{1}{2} g\hat{b}_\mu + g\hat{b}_\mu \tag{38}$$

where use has been made of the position representation of quantum mechanics, where the momenta $P_\mu = (1/i)\partial_\mu$ and the generalized momenta $B_\mu = (1/i)D_\mu$.

The resulting curvature of the $SO(4,1)_{J_{\mu\nu}\hat{b}_\mu}$-valued connection in the "QRR-gauge" is obtained by evaluating the full quantum mechanical analogue of equation (14) [where the classical gauge potentials $\Gamma^R_\mu(x)$ are replaced with their operator equivalents $(\Gamma^R_\mu(x))^{\mathrm{op}}$]. Therefore, using equation (35b), one obtains [where the prime signifies the quantum mechanical analogues of the corresponding quantities in equations (14) and (15) of the previous section]

$$[B_\mu, B_\nu] = g\mathcal{R}'^R_{\mu\nu} = gQ'^l_{\mu\nu} + gS'^t_{\mu\nu} \tag{39}$$

where

$$Q'^{l}_{\mu\nu} \equiv R'^{l}_{\mu\nu} + g[\hat{b}_\mu, \hat{b}_\nu] \tag{40a}$$

$$R'^{l}_{\mu\nu} \equiv [P_\mu, \tfrac{1}{2}\hat{b}_\nu] + [\tfrac{1}{2}\hat{b}_\mu, P_\nu] + g[\tfrac{1}{2}\hat{b}_\mu, \tfrac{1}{2}\hat{b}_\nu] \tag{40b}$$

$$S'^{t}_{\mu\nu} \equiv [P_\mu, \hat{b}_\nu] + [\hat{b}_\mu, P_\nu] + g[\tfrac{1}{2}\hat{b}_\mu, \hat{b}_\nu] + g[\hat{b}_\mu, \tfrac{1}{2}\hat{b}_\nu] \tag{40c}$$

Using the commutation relations of the Poincaré group $\mathscr{P}_{J_{\mu\nu}\hat{P}_\mu}$, where $\hat{P}_\mu = P_\mu M^{-1}$ $(=M^{-1}P_\mu)$:

$$[J_{\mu\nu}, J_{\rho\sigma}] = -i(g_{\mu\rho}J_{\nu\sigma} + g_{\nu\sigma}J_{\mu\rho} - g_{\mu\sigma}J_{\nu\rho} - g_{\nu\rho}J_{\mu\sigma}) \tag{41a}$$

$$[J_{\rho\sigma}, \hat{P}_\mu] = i(g_{\mu\sigma}\hat{P}_\rho - g_{\mu\rho}\hat{P}_\sigma) \tag{41b}$$

$$[\hat{P}_\mu, \hat{P}_\nu] = 0 \tag{41c}$$

and the dimensionless form of Finkelstein's center operator given by equation (32b), one has that

$$[\hat{b}_\mu, P_\nu] = -i(g_{\mu\nu} - \hat{P}_\mu\hat{P}_\nu)M \tag{42}$$

Using this result gives the following simplifications of (40a)-(40c):

$$Q'^{l}_{\mu\nu} \equiv R'^{l}_{\mu\nu} + g[\hat{b}_\mu, \hat{b}_\nu] \tag{43a}$$

$$R'^{l}_{\mu\nu} \equiv \tfrac{1}{4}g[\hat{b}_\mu, \hat{b}_\nu] \tag{43b}$$

$$S'^{t}_{\mu\nu} \equiv \tfrac{1}{2}g[\hat{b}_\mu, \hat{b}_\nu] + \tfrac{1}{2}g[\hat{b}_\mu, \hat{b}_\nu] \tag{43c}$$

Finally, applying the commutation relations of the Hermitian representation of $SO(4, 1)_{J_{\mu\nu}\hat{b}_\mu}$ given in equation (33c) yields

$$[B_\mu, B_\nu] = i\tfrac{9}{4}g^2 J_{\mu\nu} \tag{44}$$

where $J_{\mu\nu}$ has the quantum mechanical interpretation of generating physical angular momentum and is composed entirely of gauge potential "self-interaction" terms given in (43a)-(43c). Therefore we see that for the de Sitter connection in the "QRR-gauge," the generators of physical angular momentum take on a geometrical significance in that they represent the curvature field of the $SO(4, 1)_{J_{\mu\nu}\hat{b}_\mu}$-valued connection as demonstrated by equation (44).

The $SO(4, 1)_{J_{\mu\nu}\hat{b}_\mu}$ strong coupling constant g determines the range of the "self-interaction" terms of (43a)-(43c) and should therefore be related with the radius of the micro-de Sitter space, which is assumed to serve as the "confinement distance." The connection of g with the de Sitter space radius may be obtained by considering the following dimensional argument. We have seen that the translational components $\Gamma^{4i}_\mu(x)$ characterize a local change of the observable Lorentz coordinates when a shift of position is made in the μ direction and therefore should be interpreted as proportional to the tetrads of space-time:

$$\Gamma^{4i}_\mu(x) = gh^i_\mu(x) \tag{45}$$

where g is a constant with the dimension of inverse length. The generators of the de Sitter rotations J_{4i} and the corresponding generators of translation π_i are related by

$$\pi_i \equiv \frac{1}{R} J_{4i} \tag{46}$$

where R is the radius of the micro-de Sitter space. Therefore

$$\Gamma^{4i}_{\mu}(x) J_{4i} = h^i_{\mu}(x) \pi_i \tag{47}$$

and we see that dimensionally g may be identified with the inverse de Sitter radius, i.e.,

$$g = 1/R \tag{48}$$

up to a multiplication constant.

We now make the following ansatz in the "QRR-gauge":

$$g = -\tfrac{2}{3}\lambda \tag{49}$$

where $\lambda = 1/R$ is the inverse radius of a micro-de Sitter space and characterizes the curvature of the fiber which, as a consequence of the Cartan nature of the bundle (i.e., the soldering mechanism), may be measured in terms of centimeters. The constant, $-\frac{2}{3}$, has been introduced so that the $SO(4,1)_{J_{\mu\nu}\hat{b}_{\mu}}$ horizontal lift (covariant derivative) becomes[11] (Bohm, 1966, 1968)

$$B_{\mu} = P_{\mu} - \lambda \hat{b}_{\mu} \tag{50}$$

where

$$\hat{b}_{\mu} = \tfrac{1}{2}\{J_{\mu\sigma}, \hat{P}^{\sigma}\} \tag{51}$$

which agrees with the form of the generalized "dynamical momenta" operator used in Aldinger *et al.* (1983) to construct the model of the QRR. Using equation (49), one obtains for the commutation relation (44)

$$[B_{\mu}, B_{\nu}] = i\lambda^2 J_{\mu\nu} \tag{52a}$$

which, along with the relations

$$[J_{\rho\sigma}, B_{\mu}] = i(g_{\mu\sigma} B_{\rho} - g_{\mu\rho} B_{\sigma}) \tag{52b}$$

$$[J_{\mu\nu}, J_{\rho\sigma}] = -i(g_{\mu\rho} J_{\nu\sigma} + g_{\nu\sigma} J_{\mu\rho} - g_{\mu\sigma} J_{\nu\rho} - g_{\nu\rho} J_{\mu\sigma}) \tag{52c}$$

shows that the B_{μ} and $J_{\mu\nu}$ generate another representation of $SO(4,1)_{J_{\mu\nu}B_{\mu}}$, which turns out to play the central role in the development of the QRR, as explained in the following section.

[11]This operator was first introduced in its present context in Bohm (1966). Similar formulas have appeared in the literature, but where B_{μ} had an entirely different meaning of a center position, and when it was realized that they are related to an $SO(4, 1)$, the de Sitter space was chosen to have the radius of the universe.

Also note that the "dynamical momenta" B_μ, satisfy the quantum analogue of the kinematic constraint given by (17).

5. THE QRR AND THE INFINITE-CURVATURE LIMIT OF THE DE SITTER FIBER

The central role that the $SO(4,1)_{J_{\mu\nu}b_\mu}$ representation given by (52) plays for the model of the QRR is that in the same way that the quantum relativistic mass point (elementary point particle) is characterized by the eigenvalues m^2 of the second-order Casimir operator $P_\mu P^\mu$ of the Poincaré group, the QRR is characterized by the eigenvalues $\lambda^2\alpha^2$ of the $SO(4,1)_{J_{\mu\nu}B_\mu}$ second-order Casimir operator:

$$\lambda^2 Q = B_\mu B^\mu - \frac{\lambda^2}{2} J_{\mu\nu}J^{\mu\nu} \overset{\text{irrep}}{=} \lambda^2\alpha^2 \tag{53}$$

where the "dynamical momenta" B_μ supply a coupling between the external translational motions and the associated internal motions (which take place along the de Sitter fibers) and has the form

$$B_\mu = P_\mu - \lambda\hat{b}_\mu \tag{54a}$$

$$= P_\mu - \frac{\lambda}{2}\{J_{\mu\rho}, \hat{P}^\rho\} \tag{54b}$$

Substituting (54b) into (53) yields the following alternate form of the $SO(4,1)_{J_{\mu\nu}B_\mu}$ second-order Casimir operator:

$$\lambda^2 Q = P_\mu P^\mu + \tfrac{9}{4}\lambda^2 - \lambda^2\hat{W} \overset{\text{irrep}}{=} \lambda^2\alpha^2 \tag{55}$$

where $\hat{W} = (P_\mu P^\mu)^{-1} W$ with $W \equiv -w_\mu w^\mu$, $w_\mu = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}P^\nu J^{\rho\sigma}$.

The QRR Hamiltonian is obtained by replacing the constraint for a quantum relativistic mass point

$$\Phi \equiv P_\mu P^\mu - m^2 \approx 0 \tag{56}$$

with the QRR constraint relation

$$\Phi \equiv P_\mu P^\mu + \tfrac{9}{4}\lambda^2 - \lambda^2\hat{W} - \lambda^2\alpha^2 \approx 0 \tag{57}$$

The ≈ 0 signifies "set weakly equal to zero," since the constraint has nonvanishing commutators and one must evaluate all commutation relations prior to imposing the constraint. Following the rules of constrained Hamiltonian mechanics (Dirac, 1950), one obtains the following for the QRR Hamiltonian:

$$\mathcal{H} = \phi\Phi \equiv \phi(P_\mu P^\mu + \tfrac{9}{4}\lambda^2 - \lambda^2\hat{W} - \lambda^2\alpha^2) \tag{58}$$

where ϕ is a velocity parameter determined to be $\phi = -1/2M$ in the timelike center-of-mass gauge (Aldinger, 1985).

The constraint relation (57) taken between the canonical basis vectors $|pss_3\rangle$ [which form a basis of the space of physical states (Bohm *et al.*, 1983)] leads to the (rotator-like) mass-spin trajectory relation:

$$m^2 = \lambda^2(\alpha^2 - \tfrac{9}{4}) + \lambda^2 s(s+1) \tag{59}$$

where, for the special case of the Majorana representation, each spin occurs exactly once and has either the spectrum $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$ or $s = 0, 1, 2, \ldots$ and occurs with alternating parity. The QRR is composed of distinct physical systems (individual rotators) each of which is characterized by a particular value of $\lambda^2\alpha^2$ [the eigenvalue of the $SO(4,1)$ invariant operator $\lambda^2 Q$], where α is a continuous parameter, which, for the principal series representation of $SO(4,1)$, can take on values such that $\alpha^2 \geqslant \frac{9}{4} - s(s+1)$. In this way each hadron is considered to be a different level of a particular rotator of the QRR, where a single rotator (characterized by a specific value of α) consists of a tower of spin levels (where each level is a different state, i.e., hadron, of the physical system) with corresponding masses given by (59).

An empirical value for λ^2 (where $R = 1/\lambda$ is the radius of the de Sitter space and whose value is the same for all rotator towers) may be determined from the fits of the ρ-, ω-, or K-meson towers (for example) and is found to be $\lambda^2 \approx 0.3\ (\mathrm{GeV})^2$, which leads to a micro-de Sitter space radius of $R \approx \frac{1}{3} \times 10^{-13}$ cm (Aldinger *et al.*, 1984).

The QRR is considered to be a one-dimensionally extended object (where the extension is characterized by the radius of the de Sitter space) capable of performing translations and rotations in Minkowski space. As a consequence of the form of the $SO(4,1)_{J_{\mu\nu}B_\mu}$ "dynamical momenta" given in (54), we see that the Poincaré group $\mathcal{P}_{J_{\mu\nu}P_\mu}$ and the de Sitter $SO(4,1)_{J_{\mu\nu}B_\mu}$ are related by an Inönü-Wigner (1953) contraction process [carried out with respect to the Lorentz stability subgroup $SO(3,1)_{J_{\mu\nu}}$] in the infinite (de Sitter space)-radius limit, where $1/R = \lambda \to 0$, i.e., when the (compensating gauge operators $\hat{b}_\mu(x)$ are "turned off." In this limiting process (when the internal structure of the model is ignored), the "dynamical momenta" are

$$B_\mu = P_\mu - \lambda \hat{b}_\mu \xrightarrow{\lambda \to 0} P_\mu \tag{60}$$

and

$$[B_\mu, B_\nu] = i\lambda^2 J_{\mu\nu} \xrightarrow{\lambda \to 0} [P_\mu, P_\nu] = 0 \tag{61}$$

$$[J_{\mu\nu}, B_\rho] = i(g_{\nu\rho}B_\mu - g_{\mu\rho}B_\nu) \xrightarrow{\lambda \to 0} [J_{\mu\nu}, P_\rho] = i(g_{\nu\rho}P_\mu - g_{\mu\rho}P_\nu) \tag{62}$$

while the commutation relation of $[J_{\mu\nu}, J_{\rho\sigma}]$, equation (52c), remains unaltered and therefore the de Sitter $SO(4,1)_{J_{\mu\nu}B_\mu}$ contracts into the Poincaré group $\mathcal{P}_{J_{\mu\nu}P_\mu}$.

In order to obtain a faithful representation in the $\lambda \to 0$ contraction limit, one must allow the continuous parameter $\alpha \to \infty$ in such a way that $\lambda^2\alpha^2 \to m^2$, where $m^2 \geq 0$. Therefore the $SO(4,1)_{J_{\mu\nu}B_\mu}$ second-order Casimir operator reduces according to

$$P_\mu P^\mu + \tfrac{9}{4}\lambda^2 - \lambda^2 \hat{W} \overset{\text{irrep}}{=} \lambda^2\alpha^2 \xrightarrow{\lambda \to 0} P_\mu P^\mu \overset{\text{irrep}}{=} m^2 \tag{63}$$

(where the square of the momentum decouples from the spin) and, accordingly, the QRR Hamiltonian, equation (58), reduces in this limit to the Hamiltonian of the quantum relativistic (structureless) mass point (Aldinger *et al.*, 1984). One can conclude that in the infinite-curvature limit of the de Sitter fiber, the gauge operators $\hat{b}_\mu(x)$ are "turned off" by taking $\lambda \to 0$, leading to the well-known dynamics of the quantum relativistic mass point (elementary particle) described by the irreducible representations of the Poincaré group.

One may physically characterize the additional degrees of freedom associated with the hadron's internal space (de Sitter fibers) by saying that the "dynamical momenta" B_μ [which are related to the translational degrees of freedom contained in the vector subspace of $SO(4,1)_{J_{\mu\nu}\hat{b}_\mu}$ spanned by the gauge operators $\hat{b}_\mu(x)$] are members of a set of generators of the ten-parameter de Sitter group $SO(4,1)_{J_{\mu\nu}B_\mu}$, which allow generalized translational gauge degrees of freedom, which in the infinite de Sitter limit $R \to \infty$ (or, equivalently, $\lambda \to 0$) correspond to the Abelian generators of translations P_μ. However, due to the smallness of the de Sitter space radius [which is taken to be the confinement distance that constrains the action of the short-range self-interaction gauge operators $\hat{b}_\mu(x)$], the additional translational degrees of freedom associated with the $SO(4,1)_{J_{\mu\nu}B_\nu}$ B_μ's should not be considered as being physically detectable.

ACKNOWLEDGMENTS

I thank Prof. Arno Bohm for helpful discussions pertaining to the present paper. I also thank the Department of Physics of the University of Texas at Austin for its hospitality during the summer of 1985, where the final draft of this paper was completed. This work was supported by a research grant from Eastern Illinois University, which is gratefully acknowledged.

REFERENCES

Aldinger, R. R. (1985). *Physical Review D*, **32**, 1503.

Aldinger, R. R., Bohm, A., Kielanowski, P., Loewe, M., Magnollay, P., Mukunda, N., Drechsler, W., and Komy, S. R. (1983). *Physical Review D*, **28**, 3020.

Aldinger, R. R., Bohm, A., Kielanowski, P., Loewe, M., and Moylan, P. (1984). *Physical Review D*, **29**, 2828.

Barut, A. O., and Bohm, A. (1965). *Physical Review*, **139**, B1107.

Bohm, A. (1966). In *Lectures in Theoretical Physics*, Vol. 9B, A. O. Barut, ed., Gordon and Breach.

Bohm, A. (1968). *Physical Review*, **175**, 1767.

Bohm, A. (1979). University of Texas preprint 3992-385.

Bohm, A., Loewe, M., Biedenharn, L. C., and van Dam, H. (1983). *Physical Review D*, **28**, 3032.

Chodos, A., Jaffe, R. L., Johnson, K., Thorn, C. B., and Weisskopf, V. (1974). *Physical Review D*, **9**, 3471.

Dirac, P. A. M. (1950). *Canadian Journal of Mathemematics*, **2**, 129.

Drechsler, W. (1975). *Fortschritte der Physik*, **23**, 607.

Drechsler, W. (1977a). *Foundations of Physics*, **7**, 629.

Drechsler, W. (1977b). *Journal of Mathematical Physics*, **18**, 1358.

Drechsler, W., and Mayer, M.E. (1977). In *Fiber Bundle Techniques in Gauge Theories*, A. Bohm and J. Dollard, eds., Springer, New York.

Finkelstein, R. J. (1949). *Physical Review*, **75**, 1079.

Gell-Mann, M. (1962). *Physical Review*, **125**, 1067.

Inönü, E., and Wigner, E. P. (1953). *Proceedings of the National Academy of Sciences of the United States of America*, **39**, 510.

Kobayashi, S., and Nomizu, K. (1963). *Foundations on Differential Geometry*. Vol. I, Interscience, New York.

Marciano, W., and Pagels, H. (1978). *Physical Review*, **36C**, 137.

Salam, A., and Strathdee, J. (1977). *Physical Review D*, **16**, 2668.

Salam, A., and Strathdee, J. (1978). *Physical Review D*, **18**, 4596.

Smrz, P. K. (1977). *Progress of Theoretical Physics*, **57**, 1771.

Smrz, P. K. (1983). *Nuovo Cimento Letters*, **38**, 141.

Staunton, L. P. (1976). *Physical Review D*, **13**, 3269.

Trautman, A. (1970). *Reports on Mathematical Physics*, **1**, 29.

Wheeler, J.A. (1962). *Geometrodynamics*, Academic Press, New York.

Wigner, E. P. (1939). *Annals of Mathematics*, **40**, 149.

Yang, C. N., and Mills, R. L. (1954). *Physical Review*, **96**, 191.